Worksheet 11 **Name:** ____________________________

Week 11: 2024-11-05

Math 294: Fall 2024

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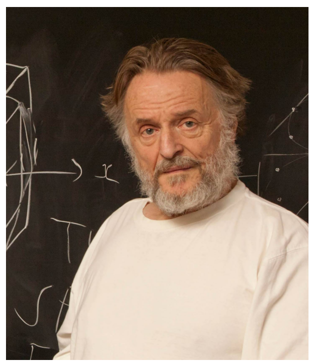
Games!

John Conway (1937-2020) was an English mathematician who studied group theory, knot theory, game theory, and coding theory. Beyond his academic contributions, he is also remembered as a popularizer of recreational mathematics; one of his most famous inventions is the so-called Conway's Game of Life. His contributions to knot theory enabled him to complete the classification of knots with up to 11 crossings (we will talk about knot theory next week!). He made significant progress on Waring's Conjecture, studying how one can express numbers as sums of fifth powers. Conway passed away due to COVID-19 in 2020.



Problem 1 (Subtraction game). Rules: Place 8 tokens in a pile. On your turn, you are allowed to take one, two, or three tokens from the pile. Once all tokens have been taken, the player who made the last move is the winner.

The parameter for the starting position is the number of tokens in the pile n. Given n, is there a way to characterize who has a winning strategy for the game where the initial pile size is n?

Problem 2 (Nim). Rules: Make two piles of 8 tokens each. On your turn, you are allowed to choose one pile and then remove any (non-zero) number of tokens from that pile. Once all tokens have been taken, the player who made the last move is the winner.

The parameters for the starting position are the sizes n and m of the two piles. Is there a way to categorize who has a winning strategy?

Problem 3 (Up-right). Rules: This game is played with a single token on a standard 8×8 chessboard. The token starts at the bottom-left corner of the board. On your turn, you may move the token either one space up, one space right, or one space diagonally up-right. The player who moves the piece into the top-right corner of the board wins.

The parameter for the starting position is the board size n. Is there a way to categorize who has a winning strategy given that we are playing on an $n \times n$ board?

Problem 4 (Three-pile Nim). Rules: Same as regular Nim, except we start with three piles instead of two. In general, we can actually start with k piles.

The analysis gets much harder, but it's still fun to think about!

Problem 5 (Double up-right). Rules: We play two copies of the Up-Right Game simultaneously. On your turn, you choose one of the boards, and make a normal Up-Right Game move on that board. However, if one of the boards is completed (the token has reached the top-right square), you must make a move on the other board, and if both boards are completed and it is your turn, you lose.

Problem 6 (Nim + up-right). Rules: We play the Up-Right Game simultaneously with one-pile Nim. On your turn, you may either make a move in the Up-Right Game, or remove any (non-zero) number of tokens from the Nim pile.