Worksheet 9 **Name:** ______________________________
Week 9: 2024-10-22
Math 294: Fall 2024
Instructor: Theo Sandstrom

We've seen sets; that's about half of math. Now time for the other half: functions! As always, work together, and ask lots of questions, both of each other, and of me!

Évariste Galois (1811-1832) was a French mathematician famous for his contributions to algebra. Born into an educated family, he took an interest in mathematics at the age of 14; by 15, he was reading original research of Lagrange. Around 1829, at age 18, Galois began making fundamental discoveries about polynomial equations. His insights into the symmetries of the solution sets to such equations laid the foundation for the field known today as group theory, and the theory of number fields. The theory that he developed helped to solve numerous questions ranging form hundreds to thousands of years old: solving polynomials by radicals (is there a "universal" quadratic formula that works for polynomials of higher degree?); squaring the circle (is it possible, using only straightedge and compass, to construct a square with the same area as a given circle?), trisecting an angle (using straightedge and compass, is it possible to cut an angle into three equal sized parts?). Galois was killed at age 21 in a duel, perhaps surrounding a broken love affair.

Problem 1. Is the set of odd integers under addition a group?

Problem 2. Is the set of integers under subtraction a group?

Problem 3. Do the symmetries of the triangle form an *abelian* group (Note: this group is denoted D_3; the dihedral group on 3 vertices.)?

Problem 4. How many symmetries does a square have? Identify them all, and give a *multiplication table* showing how the symmetries compose. What patterns do you notice in the multiplication table? (Note: this group is denoted D_4.)

Problem 5. Can you find any *subgroups* of the dihedral groups D_3 and D_4?

Problem 6. Let G be a group.

(a) Prove that the identity element is unique; that is, there is exactly one element $e \in G$ such that $eg = ge = g$ for all $g \in G$.

(b) Prove that inverses are unique; that is, for each $g \in G$, there exists exactly one element $h \in G$ so that $gh = hg = e$. (Note: since there is exactly one such element, we can denote it by g^{-1}).

Problem 7. Let G be a group, and let $a, b, c \in G$. Show that if $ac = bc$, then $a = b$.

Problem 8. Given below is a partial multiplication table for a group. Fill in the blank entries:

	e	a	b	c	d
e	e				
a		b			e
b		c	d	e	
c		d		a	b
d					