Worksheet 1 **Name:** ______________________________
Week 1: 2024-08-27
Math 294: Fall 2024
Instructor: Theo Sandstrom

This worksheet is to be completed during class on Tuesday and Thursday ***in groups***. The group work is critical! You should not be working on a different problem than your group-mates, and you should make sure that everybody in your group understands the solution before you move on. Math is a collaborative effort, and communicating math is a critical skill to succeed in the field!

I expect proof-writing to be a new skill for you. ***Don't worry!*** Over the coming weeks, we will have lots of practice together in class, and the problems will start to make more sense. Also, come to office hours! It's a great way to meet new friend/collaborators, get one-on-one guidance, or just hang out! (I am also happy to help with LaTeX questions, or questions about the Lean proof assistant.)

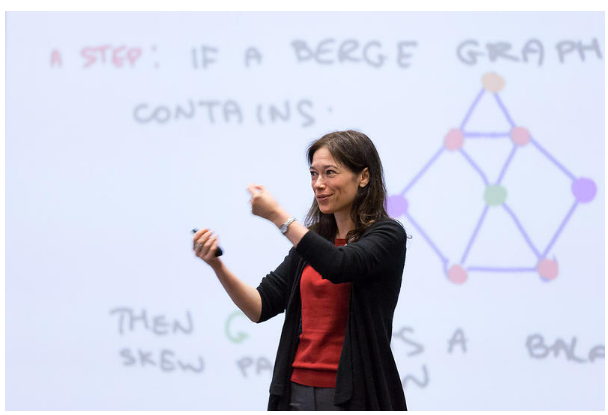
"What I have always liked about mathematics is that everything can be explained from 'first principles.' If you understand something, you really understand it all the way through; it's all in your head, and you can always go and check it."

Prof. Maria Chudnovsky (b. 1977) works in graph theory at Princeton University. She grew up in Russia and attended the Technion for her undergraduate studies. In 2003, she earned her PhD at Princeton.

"As I studied more mathematics over the next ten years, the problems got harder, the lectures got more complicated, but the feeling that there is nothing better I could possibly do with my time is still there."

Prof. Chudnovsky is known for fundamental breakthroughs in structural graph theory, especially in graph-coloring problems. Her most significant result is a proof of the strong perfect graph theorem, widely regarded as a paradigm-shift in the field.

"I really do not like giving advice, especially to such a large and heterogeneous group of people... But maybe this: if you think math is what you want to do, give it a chance."
— From a 2005 interview with the Clay Mathematics Institute



Prof. Chudnovsky, presenting her research

Problem 1. Prove that the sum of two odd numbers is an even number.

Problem 2. Prove that the square of an even number is an even number.

Problem 3. Prove that the square of an odd number is an odd number.

Problem 4 (Challenge). Prove that every odd square is one more than a multiple of 8.

Problem 5. Prove that $|xy| = |x||y|$ for any real numbers x and y.

Problem 6. If a and b are integers, we say that a *divides* b if there exists an integer q so that $b = aq$. Prove that if a divides b and b divides c, then a divides c. (*Remark:* this can be summarized by saying that divisibility is a *transitive* relation.)

Problem 7. Prove that an integer is divisible by 3 if and only if the sum of its digits is divisible by 3.

Problem 8. Suppose that $a < b$. Prove that $a < \frac{a+b}{2} < b$.

Problem 9. Let a and b be real numbers. Prove that $|a + b| \leqslant |a| + |b|$.